

CHAPTER 5

Systems of Linear Equations

What You've Learned

- In Chapter 3, you completed input-output tables and evaluated functions.
- You identified linear functions, both discrete and continuous.
- In Chapter 4, you graphed linear functions and wrote rules for linear functions.



Check Your Readiness

Functions

Use the function rule $y = 2x + 5$. Find each output.

1. $x = 0$ 2. $x = 2$ 3. $x = -3$ 4. $x = 15$

Graphing Linear Functions

Graph each linear function.

5. $y = 2x + 3$ 6. $y = \frac{1}{2}x - 2$ 7. $y = -3x + 4$

Writing Rules for Linear Functions

8. Mr. Jacobs receives a weekly salary of \$400, plus a commission of \$1,300 on each car that he sells. Write a function rule relating his weekly pay p to the number of cars he sells c .
9. Water flows from a shower head at a rate of 3 gallons per minute. Write a function rule relating the number of gallons of water g that flows from the shower head to the number of minutes m that have passed.
10. A jewelry maker sells necklaces at a craft fair. She pays the craft fair organizer \$35 to rent a table and \$1.50 for each necklace she sells. Write a function rule to represent the jewelry maker's total payment p to the craft fair organizer as a function of the number of necklaces n she sells.

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For Exercises	See Lesson
1-4	3-2
5-7	4-2
8-10	4-3



What You'll Learn Next

- In this chapter, you will learn to solve a system of linear equations by graphing.
- You will use the substitution method and the elimination method to solve systems of linear equations.
- Applying what you learn, you will use a system of equations to compare the costs of two different mobile phone plans.

Key Vocabulary

- elimination method (p. 158)
- solution of a system (p. 148)
- substitution method (p. 153)
- system of equations (p.148)

5-1

Solving Systems by Graphing

Check Skills You'll Need

- Vocabulary Review**
A ? is a rule that assigns to each input exactly one output.
- Graph each linear function.
 $y = 3x - 2$
 $y = -\frac{1}{2}x + 2$
 $y = \frac{2}{3}x + 1$

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Lesson 4-2

What You'll Learn

To solve systems of two linear equations in two variables by graphing the equations

New Vocabulary solution of a system, system of equations

Why Learn This?

Graphing systems of equations can help you compare different rental options.

A **system of equations** is a set of two or more equations that have the same variables. The **solution of a system** is any ordered pair that satisfies all equations in the system.



One method of solving a system of linear equations is to graph each equation and find any intersection points.

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8.EE.8.a, 8.EE.8.b, 8.EE.8.c

EXAMPLE Solving a System by Graphing

- Solve the system by graphing. $y = 2x - 1$
 $y = -x + 5$

Graph both equations in the same coordinate plane.

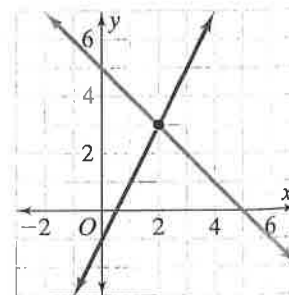
$y = 2x - 1$ ← The slope is 2. The y-intercept is -1.

$y = -x + 5$ ← The slope is -1. The y-intercept is 5.

The lines appear to intersect at (2, 3). Check by replacing x with 2 and y with 3 in each equation.

$$\begin{array}{ll} y = 2x - 1 & y = -x + 5 \\ 3 \stackrel{?}{=} 2(2) - 1 & 3 \stackrel{?}{=} -(2) + 5 \\ 3 = 3 \checkmark & 3 = 3 \checkmark \end{array}$$

The solution of the system is (2, 3).



Vocabulary Tip

The lines intersect at one point. This means that the system of equations has one solution.

Quick Check

- Solve the system of equations by graphing. $y = 2x - 4$
 Check the solution. $y = -\frac{1}{2}x + 1$

You can also use tables when graphing systems of equations.

EXAMPLE Solving a System by Graphing

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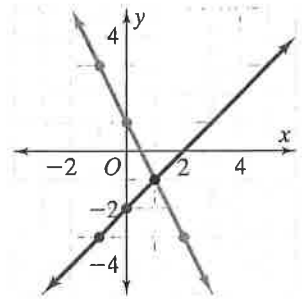
For help graphing a line by making a table, go to Lesson 4-2.

- 2 Solve the system by graphing. $x - y = 2$
 $2x + y = 1$

Make a table for each equation in the system.

$x - y = 2$	
x	y
-1	-3
0	-2
1	-1

$2x + y = 1$	
x	y
-1	3
0	1
1	-1



Notice that $(1, -1)$ is an ordered pair in both tables.

Graph both equations in the same coordinate plane.

The lines intersect at $(1, -1)$.

So the solution of the system is $(1, -1)$.

Quick Check

2. Solve the system by graphing. $-x + y = -5$
 $2x + y = 4$

EXAMPLE Application: Comparison Shopping



- 3 You need to rent a video camera. Video Barn charges a \$30 rental fee plus \$35 per day. Allied Rental charges a \$45 rental fee plus \$30 per day. Which company should you choose? Justify your answer.

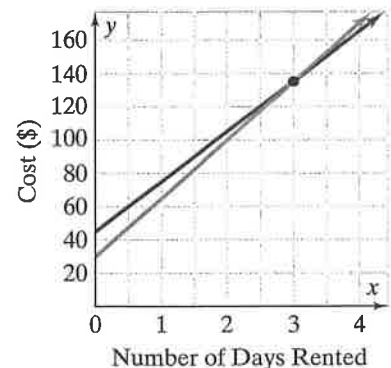
Step 1 Write a system of equations to represent the situation. Let x = the number of days and y = the total cost.

$$y = 30 + 35x \quad \leftarrow \text{The slope is 35. The y-intercept is 30.}$$

$$y = 45 + 30x \quad \leftarrow \text{The slope is 30. The y-intercept is 45.}$$

Step 2 Graph both equations in the same coordinate plane. The lines appear to intersect at $(3, 135)$.

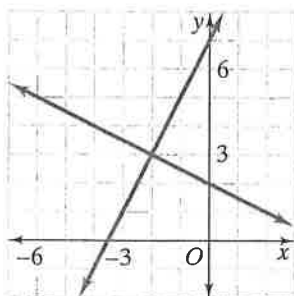
Step 3 Analyze the graph. For a rental of up to 3 days, Video Barn is less expensive. For a 3-day rental, both companies charge \$135. For rentals longer than 3 days, Allied Rental is less expensive.



Quick Check

3. To rent scooters, Sam's charges a \$30 fee plus \$8 per hr. Rosie's charges a \$20 fee plus \$10 per hr. Which company should the Morris family choose?

Check Your Understanding



- Vocabulary** When graphing a system of linear equations, the point where the lines intersect is the ? of the system.
- What is the solution of the linear system at the left?
- Reasoning** Can a system of two linear equations have exactly two solutions? Explain.
- Determine if $(1, 4)$ is a solution of the system.

$$y = 2x + 2$$

$$y = 4x - 6$$

Homework Exercises

For more exercises, see **Extra Skills and Word Problems**.

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For Exercises	See Examples
5–10	1
11–13	2
14, 15	3

Solve each system of equations by graphing. Check your solution.

5. $y = x + 2$

$y = 3x$

8. $y = 3x - 4$

$y = -3x + 2$

11. $x - y = 1$

$5x - 4y = 0$

6. $y = -x + 8$

$y = \frac{1}{3}x$

9. $y = x + 4$

$y = -2x - 2$

12. $y = 3x - 4$

$x + 2y = 6$

7. $y = \frac{1}{2}x + \frac{3}{2}$

$y = 2x - 6$

10. $y = \frac{4}{3}x + 3$

$y = -\frac{2}{3}x - 3$

13. $-\frac{1}{3}x + y = -2$

$x + y = -2$

- The community theater is selling tickets for their fall play. On the first day of ticket sales, the theater sold 9 adult tickets and 3 student tickets for a total of \$75. The theater collected \$67 on the second day by selling 5 adult tickets and 8 student tickets. What are the prices of one adult ticket and one student ticket?
- The sum of two numbers is 13. Their difference is 3. Write a system of equations to model this situation. Use graphing to solve the system.

GPS

16. **Guided Problem Solving** A system is made up of two lines, a and b . Line a passes through points $(2, 3)$ and $(6, 5)$. Line b passes through points $(2, 5)$ and $(4, 2)$. What is the solution of the system?
- How can you use a graph to estimate the solution of the system?
 - How will an equation of each line help you check your answer?

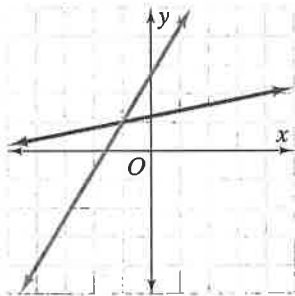
17. A system is made up of two lines, c and d . Line c passes through points $(1, 1)$ and $(4, 3)$. Line d passes through points $(2, 3)$ and $(3, 1)$. What is the solution of the system?

5-1b

Activity Lab

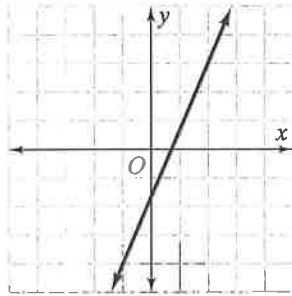
Solutions of Linear Systems

One Solution



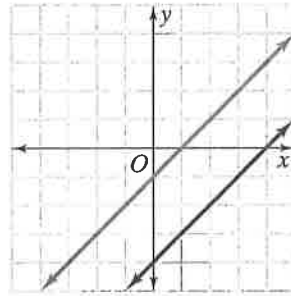
The lines have different slopes and intersect at one point.

Infinitely Many Solutions



The lines have the same slope and y-intercept. They are the same line.

No Solution



The lines have the same slope and different y-intercepts. They are parallel and do not intersect.

EXAMPLES Special Types of Linear Systems

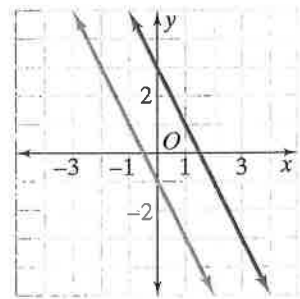
- 1 Solve the system of equations by graphing. $y = -2x + 3$
 $y = -2x - 1$

Graph both equations in the same coordinate plane.

$y = -2x + 3$ ← The slope is -2 . The y-intercept is 3 .

$y = -2x - 1$ ← The slope is -2 . The y-intercept is -1 .

The lines are parallel. The system has *no solution*.



- 2 Solve the system of equations by graphing. $2x - 2y = 4$
 $-x + y = -2$

Graph both equations in the same coordinate plane.

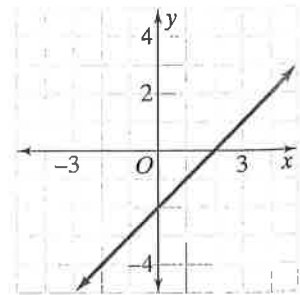
$$2x - 2y = 4$$

$$-x + y = -2$$

x	0	1	2
y	-2	-1	0

x	0	1	2
y	-2	-1	0

The equations represent the same line. The system has *infinitely many solutions*.



Exercises

Solve each system by graphing.

1. $2y = 6x + 4$
 $3x - y = -2$

2. $2x - 6y = 6$
 $3y = x + 3$

3. $y = 2x + 6$
 $y = -\frac{1}{2}x + 1$

1.
2.
S
v
b
a

5-2

Solving Systems by Substitution

 Check Skills You'll Need

1. Vocabulary Review

How can you use the *Distributive Property* to solve for x in this equation?

$$2x + 3(x + 2) = 16$$

2. Substitute $2x - 1$ for y in each equation. Solve for x .

$$y + 2x = 3$$

$$x - 2y = 8$$

$$2x + 3y = -15$$



for Help
Lesson 2-3

 CONTENT STANDARDS

8.EE.8.b, 8.EE.8.c

Vocabulary Tip

Substitution means one value or expression can be used in place of another.

What You'll Learn

To solve a system of linear equations by substitution

New Vocabulary substitution method

Why Learn This?

You can use a system of equations to compare the costs of two phone plans. To solve a system, you can solve one of the equations for one of the variables. This is called the **substitution method**.

EXAMPLE Using Substitution

- 1 Solve the system by substitution. $y = 4x - 1$
 $2x + 2y = 3$

Step 1 Because $y = 4x - 1$, substitute $4x - 1$ for y in $2x + 2y = 3$.

$$2x + 2y = 3$$

← Write the second equation.

$$2x + 2(4x - 1) = 3$$

← Substitute $4x - 1$ for y .

$$2x + 8x - 2 = 3$$

← Use the Distributive Property.

$$10x - 2 = 3$$

← Simplify.

$$10x = 5$$

← Add 2 to each side.

$$x = \frac{1}{2}$$

← Divide each side by 10.

Step 2 Substitute $\frac{1}{2}$ for x in either equation and solve for y .

$$y = 4x - 1$$

← Write either equation.

$$y = 4\left(\frac{1}{2}\right) - 1$$

← Substitute $\frac{1}{2}$ for x .

$$y = 1$$

← Simplify.

The solution of the system is $\left(\frac{1}{2}, 1\right)$.

Check Replace x with $\frac{1}{2}$ and y with 1 in each equation.

$$y = 4x - 1 \qquad 2x + 2y = 3$$

$$1 \stackrel{?}{=} 4\left(\frac{1}{2}\right) - 1 \qquad 2\left(\frac{1}{2}\right) + 2(1) \stackrel{?}{=} 3$$

$$1 = 1 \quad \checkmark \qquad 3 = 3 \quad \checkmark$$

 Quick Check

1. Solve the system by substitution.

$$y = x + 1$$

- Check your answer.

$$2x + y = -2$$

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EXAMPLE

Solving for a Variable and Using Substitution

- 2 Solve the system by substitution. $-9x + 5y = -1$
 $-3x + y = 4$

Step 1 Solve one of the equations for one of the variables.

$$\begin{aligned} -3x + y &= 4 && \leftarrow \text{Write the second equation.} \\ y &= 3x + 4 && \leftarrow \text{Add } 3x \text{ to each side.} \end{aligned}$$

Step 2 Substitute $3x + 4$ for y in the other equation and solve for x .

$$\begin{aligned} -9x + 5y &= -1 && \leftarrow \text{Write the first equation.} \\ -9x + 5(3x + 4) &= -1 && \leftarrow \text{Substitute } 3x + 4 \text{ for } y. \\ -9x + 15x + 20 &= -1 && \leftarrow \text{Use the Distributive Property.} \\ 6x &= -21 && \leftarrow \text{Simplify. Subtract } 20 \text{ from each side.} \\ x &= -3.5 && \leftarrow \text{Divide each side by } 6. \end{aligned}$$

Step 3 Substitute -3.5 for x in either equation and solve for y .

$$\begin{aligned} -3x + y &= 4 && \leftarrow \text{Write either equation.} \\ -3(-3.5) + y &= 4 && \leftarrow \text{Substitute } -3.5 \text{ for } x. \\ 10.5 + y &= 4 && \leftarrow \text{Simplify.} \\ y &= -6.5 && \leftarrow \text{Subtract } 10.5 \text{ from each side.} \end{aligned}$$

The solution of the system is $(-3.5, -6.5)$.

Quick Check

2. Solve the system by substitution. $-2x + y = 3$
Check your answer. $3x - 2y = 0$

EXAMPLE

Application: Phone Plans

- 3 Nia's phone plan costs \$42 per month plus \$.20 per text. Rick's phone plan costs \$45 per month plus \$.10 per text. For what number of texts will their plans cost the same per month? What will the monthly cost be?

Step 1 Write a system of equations. Let x = the number of texts and y = the cost per month.

$$\begin{aligned} y &= 42 + 0.2x && \leftarrow \text{Cost of Nia's plan} \\ y &= 45 + 0.1x && \leftarrow \text{Cost of Rick's plan} \end{aligned}$$

Step 2 $42 + 0.2x = 45 + 0.1x$ \leftarrow Substitute $42 + 0.2x$ for y in the second equation.

$$\begin{aligned} 0.1x &= 3 && \leftarrow \text{Subtraction Property of Equality.} \\ x &= 30 && \leftarrow \text{Divide each side by } 0.1. \end{aligned}$$

Step 3 $y = 42 + 0.2(30)$ \leftarrow Substitute 30 for x in either equation.
 $y = 48$ \leftarrow Simplify.

For 30 texts per month, both plans will cost \$48.

Test Prep Tip

Check your solution by substituting 30 for x into the other equation to see if the monthly charge is the same.

$$\begin{aligned} y &= 45 + 0.1x \\ y &= 45 + 0.1(30) \\ y &= 45 + 3 \\ y &= 48 \checkmark \end{aligned}$$

✓ Quick Check

3. Juan's phone plan costs \$40 per month plus \$.40 per text. For what number of texts will Juan's and Nia's plans cost the same per month?
 What will the monthly cost be?

● More Than One Way

Solve the linear system.

$$-x + y = 5$$

$$\frac{1}{2}x + y = 8$$

Jasmine's Method

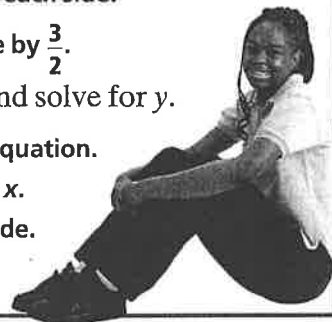
I can solve the system by substitution. First, I can solve the equation $-x + y = 5$ for y to get $y = x + 5$.

$$\begin{array}{ll} \frac{1}{2}x + y = 8 & \leftarrow \text{Write the second equation.} \\ \frac{1}{2}x + (x + 5) = 8 & \leftarrow \text{Substitute } x + 5 \text{ for } y. \\ \frac{3}{2}x + 5 = 8 & \leftarrow \text{Simplify.} \\ \frac{3}{2}x = 3 & \leftarrow \text{Subtract 5 from each side.} \\ x = 2 & \leftarrow \text{Divide each side by } \frac{3}{2}. \end{array}$$

Now I can substitute 2 for x in either equation and solve for y .

$$\begin{array}{ll} -x + y = 5 & \leftarrow \text{Write the first equation.} \\ -(2) + y = 5 & \leftarrow \text{Substitute 2 for } x. \\ y = 7 & \leftarrow \text{Add 2 to each side.} \end{array}$$

The solution of the system is $(2, 7)$.

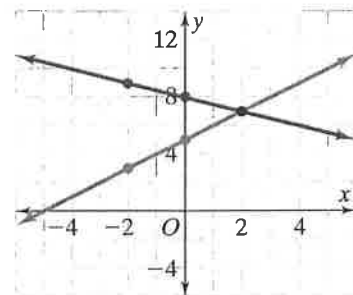


Kevin's Method

I can solve the system by graphing. I can make a table for each equation in the system. Then I can graph the lines.

$-x + y = 5$	
x	y
-5	0
0	5
5	10

$\frac{1}{2}x + y = 8$	
x	y
-2	9
0	8
2	7



The solution of the system is $(2, 7)$.

Solve the linear system.

$$-x + y = -2$$

Explain why you chose the method you used.

$$\frac{1}{3}x + y = 2$$

Check Your Understanding

1. Which of the following systems of equations would be most efficiently solved by using the substitution method? Explain.

$$\begin{array}{l} \text{System A: } 6x + 3y = -3 \\ \quad \quad \quad 4x + 2y = 2 \end{array} \qquad \begin{array}{l} \text{System B: } 4x + 3y = 11 \\ \quad \quad \quad 2x - y = 3 \end{array}$$

2. Suppose you want to solve the system at the right using substitution. What should your first step be?

$$\begin{array}{l} x - 4y = 2 \\ 3x + 5y = 40 \end{array}$$

3. Explain why you might choose to solve a system of equations using substitution rather than by graphing.

Homework Exercises

For more exercises, see **Extra Skills and Word Problems**.

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For Exercises	See Examples
4–6	1
7–9	2
10, 11	3

Solve each system of equations using substitution. Check your answer.

4. $y = -x + 8$
 $-x + 3y = 0$

5. $2x - 2y = 2$
 $y = -5x + 14$

6. $2y - x = 3$
 $y = 2x - 6$

7. $-x + y = 4$
 $-2x - y = 2$

8. $x - 2y = 2$
 $-3x + 4y = 3$

9. $2y - \frac{8}{3}x = 6$
 $y + \frac{2}{3}x = -3$

10. The cost of membership in a discount grocery club includes a monthly charge and a one-time joining fee. The total cost of membership for 6 months is \$80, and the total cost of membership for 1 year is \$110. Write and solve a system of equations to find the monthly charge and the joining fee.
11. Trey's online music club charges a monthly fee of \$20 plus \$.80 per song downloaded. Deb's online music club charges a monthly fee of \$21 plus \$.60 per song downloaded. For what number of songs downloaded will the monthly charge be the same for both clubs?



12. **Guided Problem Solving** Natasha is going to the local spring festival. The table shows the ticket options. Which ticket option should she choose? Justify your answer.

Ticket Option	Admission Price	Price per Ride
A	\$5	\$.30
B	\$3	\$.80

- **Understand the Problem** Write a system of equations that models the situation.
- **Make a Plan** How can you find the answer using your system?

13. A group of friends wants to play paintball. The pricing options are shown in the table. Which option should the friends choose? Justify your answer.

Pricing Option	Equipment Rental	Price per Game
1	\$25	\$7
2	\$40	\$4

14. **Writing in Math** What would your first step be in solving the system below? Explain your reasoning?

$$y = 2x - 4$$

$$4x - 2y = 0$$

15. **Reasoning** A community theater group put on a play last month. Twenty more adults attended than children. Adult tickets cost \$8 each and children's tickets cost \$3 each. The theater group collected \$1,260. How many adult tickets were sold?

16. A grocery store makes a 20-pound mixture of almonds and cashew nuts. The store charges \$4 per pound for almonds and \$5.50 per pound for cashews. The total value of the mixture is \$92. How many pounds of each type of nut are in the mixture?



17. **Challenge** Use substitution to solve the system of equations. Write the solution in the form (x, y, z) .

$$x + y + z = -54$$

$$x = -6y$$

$$z = 14y$$

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Test Prep and Mixed Review

Practice

Multiple Choice

18. Which of the following is the solution of the system of equations?

$$y = 3x - 8$$

$$y = 4 - x$$

- (A) (3,1) (B) (1,3) (C) (-3,1) (D) (3,-1)

19. What is the decimal expansion of $\frac{1}{6}$?

- (F) 1.6 (G) 0.6 (H) $0.1\bar{6}$ (J) $0.01\bar{6}$

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For Exercises	See Lesson
20-22	4-1

Find the slope of the line that passes through each pair of points.

20. (3,7) and (2,2) 21. (3,0) and (1,-6) 22. (-2,-1) and (-5,-2)